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SUPPLEMENT TO THE PAPER: Separating the regular and irregular energy levels and their statistics in Hamiltonian system with mixed classical dynamics[1]

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Abstract. As a technical supplement to the above mentioned paper we present 192 consecutive eigenstates for the Robnik billiard with the shape parameter $\lambda = 0.15$ from 10,001st to 10,192nd, by showing the plots in the configuration space and in the phase space. The latter is smoothed projection of the Wigner function onto the surface of section. By comparison with the classical SOS plots we thus examine all eigenstates and classify them in regular and irregular: There are 70 regular states and 122 irregular states, thus giving the estimate of the relative measure of the regular component $\rho_1 = 0.365$, which is in excellent agreement with the classical value $\rho_1 = 0.360$ calculated and reported by Prosen and Robnik (1993).

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Explanation of plots

In this work we provide additional material to our recent paper (Li and Robnik 1994a). The method and the mathematics of analyzing the eigenstates in the Robnik billiard defined as the quadratic complex conformal map of the unit disk (in the z-plane, $|z| \leq 1$) onto the complex w-plane, namely

$$\mathcal{B}_\lambda = \{w | w = z + \lambda z^2, \quad |z| \leq 1\}. \tag{1}$$

with $\lambda = 0.15$, is presented in our recent paper (Li and Robnik 1994b).

The wavefunctions we are looking at are the eigenfunctions of the Schrödinger equation (Helmholtz equation):

$$\Delta\Psi + E\Psi = 0, \qquad \Psi = 0 \quad \text{at} \quad \text{the} \quad \text{boundary} \quad \text{of} \quad \mathcal{B}_\lambda \tag{2}$$

where $E = k^2$ is the eigenenergy and k the wavenumber. (So we are using units such that Planck's constant $\hbar = 1$ and $2m = 1$, where m is the mass of the point billiard particle.)

In the configurational plots in figures 1-8 we show the contours of constant probability density for the eigenstates 10,001st through 10,192nd with each figure containing 24 consecutive plots ordered as left-right top-down sequence. The scale and the coordinates are uniquely specified by the equation defining the boundary namely, $w = z + \lambda z^2$, where $|z| = 1$. The contours of constant probability density are chosen in steps of 1/8 of the maximal value of each individual plot.

In order to investigate the eigenstates in the quantum (Wigner) phase space we have first to define the classical phase space and the surface of section. The usual bounce map (Poincaré map) in the Birkhoff coordinates (arclength versus tangent unit velocity vector component) is not suitable for our purpose, because Ψ vanishes on the boundary. Therefore we choose the surface of section defined by $v = \mathrm{Im}(w) = 0$: Our surface of section is now specified by the crossing point coordinate u on the abscissa versus the conjugate momentum equal to the tangential component of the velocity vector of length k with respect to the line of section $v = 0$. In figure 9 we show 24 identical plots of the geometry of the largest chaotic component for $\lambda = 0.15$, in exactly the same size as the quantal phase space plots which we will show in

figures 10-17, which enables the reader to perform the comparison of classical and quantal plots and the classification of the eigenstates in regular and irregular, by overlaying the sheets. We do not show further details of the KAM scenario inside the stability islands in order not to obscure the structure of the phase space.

The Wigner function (of an eigenstate $\Psi(u,v)$) defined in the full phase space (u,v,p_u,p_v) is

$$W(\mathbf{q},\mathbf{p}) = \frac{1}{(2\pi)^2}\int d^2\mathbf{X}\exp(-i\mathbf{p}\cdot\mathbf{X})\Psi(\mathbf{q}-\mathbf{X}/2)\Psi(\mathbf{q}+\mathbf{X}/2) \qquad (3)$$

where we have specialized to our real Ψ case, and also two degrees of freedom and $\hbar = 1$. Here $\mathbf{q} = (u,v)$ and $\mathbf{p} = (p_u,p_v)$. In order to compare the quantum Wigner functions with the classical Poincaré maps on the surface of section we define the following projection of (3) given as

$$\rho_{SOS}(u,p_u) = \int dp_v W(u,0,p_u,p_v), \qquad (4)$$

which nicely reduces the number of integrations by one and is equal to

$$\rho_{SOS}(u,p_u) = \frac{1}{2\pi}\int dx \exp(ixp_u)\Psi(u+\frac{x}{2},0)\Psi(u-\frac{x}{2},0) \qquad (5)$$

As is well known the Wigner function and its projections are not positive definite and indeed one typically finds small and inconvenient but nevertheless physical oscillations around zero which seriously obscure the main structural features. Therefore in order to compare the classical and quantal phase space structure it is advisable to smooth the Wigner function or its projections (5) by a normalized Gaussian kernel with a suitably adapted dispersion. Such procedure has been introduced and used in (Takahashi 1989, Leboeuf and Saraceno 1990, Heller 1991, Prosen and Robnik 1993), which is Husimi type representation but the effective area of our Gaussian kernel will be smaller than 2π.

In figures 10-17 we show the smoothed object (5) for each consecutive eigenstate from the 10,001st to the 10,192nd, each figure containing 24 states arranged in order left-right top-down. The lowest contour is at the level of

0.15 of the maximal value and the step size upwards is also 0.15 of the maximum.

The examination of the states in figures 10-17 and classification leads to the following result: There are 70 regular and 122 irregular states. This yields the relative fraction of regular levels $\rho_1 = 0.365$, in excellent agreement with the classical value $\rho_1 = 0.360$ (Prosen and Robnik 1993).

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